Girls Get
Curves

Solution Guide for Chapter 6

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 94

2. Isosceles triangles are acute triangles.

Is this always, sometimes or never true? Well, we can certainly think of an example of this (like a triangle with angles 50°, 50°, and 80° - that would be isosceles *and* acute), so the answer isn't "never." But can we think of an isosceles triangle that is not acute? Sure we can! Half of a square is an isosceles triangle with angles 45°, 45°, and 90°. And since one angle is 90°, the triangle isn't acute. So "Isocecles triangles are acute triangles" is only *sometimes* true.

Answer: sometimes

3. Isosceles triangles are equilateral.

This is *sometimes* true, like in the case of, um, equilateral triangles. But most isosceles triangles aren't equilateral, so the statement isn't *always* true.

Answer: sometimes

4. Equilateral triangles are scalene.

By definition, equilateral triangles have three congruent angles. And *by definition*, scalene triangles have three angles whose measures are all different. So this can never be true!

Answer: never

5. Scalene triangles are isosceles.

Again, by definition, isosceles triangles have at least 2 congruent angles, and the angles in scalene triangles all must be different. So this statement can never be true.

Answer: never

6. Isosceles triangles are right triangles.

Well, this is sometimes true. For example, half of a square is an isosceles triangle with angles 45°, 45°, and 90°, and so that's a right triangle. But there are plenty of isosceles triangles that don't have a right angle (like an equilateral triangle), so it's not always true.

Answer: sometimes

7. A triangle has two obtuse angles.

This can never happen, and here's why: An obtuse angle measures more than 90°, and if there were two angles that measured more than 90°, that would mean the sum of those two angles would be more than 180°, which is impossible, since all three angles of a triangle always add up to 180°. **Answer: never**

8. A triangle has three acute angles.

This is certainly true sometimes; for example, equilateral triangles have three acute angles. But this isn't always true – any obtuse triangle or right triangle does NOT have three acute angles. **Answer: sometimes**

9. A triangle has at least one acute angle.

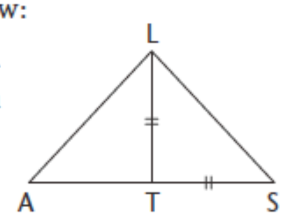
This is certainly sometimes true – equilateral triangles, for example, have at least one acute angle (in fact, they have 3!). But is it always true, or is it possible to find a counterexample? In other words, is there such thing as a triangle that has no acute angles? If a triangle had no acute angles, that would be that all three of its angles measured 90° or more. And that would mean that the sum of all three angles would end up being at least 90° + 90° + 90° = 270°! And that's impossible, since all three angles of every triangle always add up to 180°. So that means there cannot be a counterexample, and the statement "A triangle has at least one acute angle" must ALWAYS be true.

Answer: always

DTM from p. 97-98

For #2–8, fill in the blanks of the proof below:

Given: $\overline{LT} \cong \overline{TS}$, and $\overline{LT}$ is the perpendicular bisector of $\overline{AS}$. Prove that $\triangle ATL$ is a right isosceles triangle.



Proof

Statements	Reasons
1. $\overline{LT} \cong \overline{TS}$	1. (#2)
2. $\overline{LT}$ is the $\perp$ bisector of $\overline{AS}$.	2. (#3)
3. (#4)	3. If a segment is a $\perp$ bisector, then it divides a segment into two $\cong$ parts. (partial definition of $\perp$ bisector)
4. $\overline{AT} \cong \overline{LT}$	4. (#5)
5. $\triangle ATL$ is isosceles.	5. (#6) (Good progress!)
6. (#7)	6. If a segment is a $\perp$ bisector, then it creates a right angle. (partial definition of $\perp$ bisector)
7. $\therefore$ (#8)	7. If a $\triangle$ is isosceles and has a right angle, then it is a right isosceles $\triangle$.

#2 and #3 are easy – those are just the givens!

2. **Given**

3. **Given**

4. (Notice that this is actually the Statement on line "3")

Okay, this Statement must match up with the "then" part of the Reason on this line, right? And the "then" part mentions congruent segments created by a perpendicular bisector – and in our diagram, those two congruent segments would be $\overline{AT} \cong \overline{TS}$.

Answer: $\boldsymbol{\overline{AT} \cong \overline{TS}}$

5. Now we're looking for the Reason for the statement $\overline{AT} \cong \overline{LT}$, right? Hm. We just found out that $\overline{AT} \cong \overline{TS}$, and we were given $\overline{LT} \cong \overline{TS}$, so now the Transitive Property tells us that $\overline{AT} \cong \overline{LT}$. Nice!

Answer: Transitive Property (If $\boldsymbol{\overline{LT} \cong \overline{TS}}$ and $\boldsymbol{\overline{AT} \cong \overline{TS}}$, then $\boldsymbol{\overline{AT} \cong \overline{LT}}$)

6. What is the Reason for saying that $\triangle ATL$ is isosceles? Well, we have just found out that two of its sides are congruent!

Answer: If a triangle has two congruent sides, then it's isosceles

7. Okay, this Statement must match up with the "then" part of the Reason on this line, right? And the "then" part mentions a right angle created from a perpendicular bisector. So the right angle could be either $\angle LTA$ or $\angle LTS$, right? But since our goal is to prove that $\triangle ATL$ is a right isosceles triangle, let's pick the angle that is inside the triangle $\triangle ATL$, which is $\angle LTA$.

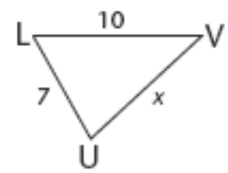
Answer: $\boldsymbol{\angle LTA}$ is a right angle

8. Finally, this is our conclusion line, which is the thing we were trying to prove all along: $\triangle ATL$ is a right isosceles triangle. And happily, this matches up with the "then" part of the Reason on this line. Done!

Answer: $\boldsymbol{\triangle ATL}$ is a right isosceles triangle

DTM from p. 102-103

2. Given: $\triangle LUV$ with sides 10, 7, and x. What is the range of possible values for x?

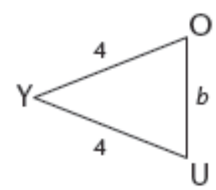


The Triangle Inequality says that, for example, $7 + x > 10$.
Solving, that gives us $\boldsymbol{x > 3}$. Another inequality we know we can write is: $7 + 10 > x$, in other words: $\boldsymbol{17 > x}$. The third inequality is the unhelpful one: $10 + x > 7$, in other words, $x > -3$. Yeah, we kinda already knew that x was bigger than a negative number…
Combining the two helpful ones into a conjunction, we get: $\boldsymbol{17 > x > 3}$. And that's the range of possible values for x!

Answer: $\boldsymbol{17 > x > 3}$

Or: $\boldsymbol{3 < x < 17}$ *(they mean the same exact thing!)*

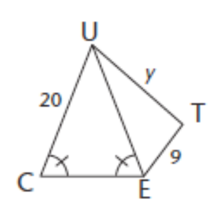
3. Given: $\triangle YOU$ is an isosceles triangle with legs of length 4. What is the range of possible values for its base, b?



Here's one inequality: $4 + 4 > b$, which gives us $\boldsymbol{b < 8}$. The next inequality is $4 + b > 4$, in other words, $b > 0$. Pretty obvious, but we'll include it in the inequality anyway. And the third inequality we can set up is $b + 4 > 4$; the same one as before, which also gives $b > 0$. (But this always happens in isosceles triangles – we always end up with two matching, "unhelpful" inequalities that tell us the base has to be greater than zero. Like, duh.)
So the full range of possible values for b is that it can be anything between 0 and 8!

Answer: $\boldsymbol{0 < b < 8}$

4. In the CUTE diagram, what is the length of $\overline{UE}$, and how do we know? Given this, what is the possible range of values for y?

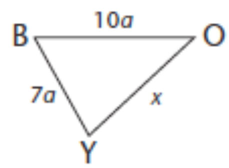


Since the diagram markings tell us that $\angle UCE \cong \angle UEC$, we also know that the segments opposite those angles must be congruent – it's an isosceles triangle, after all! So that means $\boldsymbol{UE = 20}$. Now we can shift our attention to the triangle on the right, $\triangle EUT$, whose sides are 20, y, and 9. So the Triangle Inequality tells us that $9 + y > 20$, which simplifies to $\boldsymbol{y > 11}$. The Triangle Inequality also tells us that $20 + y > 9$, in other words, $y > -11$. (Not helpful.) And thirdly, the Triangle Inequality tells us that $9 + 20 > y$, in other words: $\boldsymbol{29 > y}$. Combining the two helpful ones into a conjunction, we get y's total range of possible values: $\boldsymbol{29 > y > 11}$.

Answer: $\boldsymbol{29 > y > 11}$

Or: $\boldsymbol{11 < y < 29}$ *They mean the same exact thing!*

5. Given $\triangle BOY$ with sides $10a$, $7a$, and x. What is the range of possible values for x (in terms of a)?



This is the exact same problem as #3, but the 7 and the 10 each have an extra factor of "a" multiplied times them! We won't be scared off by that little variable though, will we?

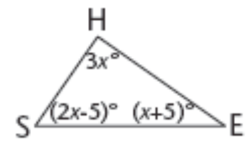
The Triangle Inequality says that $7a + x > 10a$. Solving, that gives us $\boldsymbol{x > 3a}$. Another inequality we know we can write is: $7a + 10a > x$, in other words: $\boldsymbol{17a > x}$. The third inequality is the unhelpful one: $10a + x > 7a$, in other words, $x > -3a$. Since a has to be positive (think about that for a second), we know that $-3a$ is a negative value, and yes, we already knew that x must be greater than a negative number. ☺

Combining the two helpful inequalities into a conjunction, we get: $\boldsymbol{17a > x > 3a}$. (In other words: $\boldsymbol{3a < x < 17a}$.) And that's the range of possible values for x – no matter what "a" happens to be!

Answer: $\boldsymbol{3a < x < 17a}$

DTM from p. 105-106

2. In $\triangle SHE$, $m\angle \boldsymbol{S} = (2x - 5)°$, $m\angle \boldsymbol{H} = 3x°$, and $m\angle \boldsymbol{E} = (x + 5)°$. Is $\triangle SHE$ acute, obtuse, or right? *(Hint: First solve for x.)*



How do we solve for x? Well, since we know that the three angles in a triangle always add up to 180, we can just set up this equation and solve:

$(2x - 5) + 3x + (x + 5) = 180$

→ $2x - 5 + 3x + x + 5 = 180$

→ $6x - 5 + 5 = 180$

→ $6x = 180$

→ $\boldsymbol{x = 30}$

Now that we have x, we can figure out the angles' measurements! They are:

$m\angle S = (2x - 5)° = (2 \cdot \mathbf{30} - 5)° = (60 - 5)° = (60 - 5)° = \underline{\mathbf{55}}\mathbf{°}$

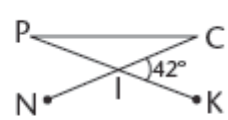
$m\angle H = 3x° = 3(\mathbf{30})° = \underline{\mathbf{90}}\mathbf{°}$

$m\angle E = (x + 5)° = (\mathbf{30} + 5) = \underline{\mathbf{35}}\mathbf{°}$

And yep, 55 + 90 + 35 = 180, so it seems we didn't make any mistakes, which is always good. But wait, what did the problem want? It wants to know if this triangle is acute, right, or obtuse. Well, since it has one angle that equals 90°, it's a right triangle!

Answer: It's a right triangle

3. In the PICNK diagram, $\triangle PIC$ is an isosceles triangle with base $\overline{PC}$, and $\angle \boldsymbol{CIK} = 42°$. What are the measurements for the three interior angles of $\triangle PIC$? (*Hint: First label $\angle \boldsymbol{P}$ and $\angle \boldsymbol{C}$ with n°, and think about supplementary angles!*)



So we're supposed to find the measurements of $\angle P, \angle C$ & $\angle PIC$, right? (Notice we can't call the triangle's middle angle $\angle I$, because I is the vertex for more than one angle!)

Well, what do we know already? If $\triangle PIC$ is isosceles with base $\overline{PC}$, that means $\angle P \cong \angle C$, right? And the Hint says to call those angles' measurements $n°$. Okay. Now notice that $\angle PIC$ is supplementary to the angle $\angle CIK$; they add up to create a straight angle. Since we are told that $\angle CIK = \mathbf{42°}$, we can write:

$\angle PIC + \angle CIK = 180°$

→ $\angle PIC + \mathbf{42°} = 180°$

→ $\angle PIC = 138°$

Great! Now, how do we find $\angle P$ & $\angle C$? Well, all three angles in a triangle add up to 180°, and we can fill in one of the angles now, so we can write down this equation:

$$\angle PIC + \angle P + \angle C = 180°$$

$$\rightarrow \mathbf{138° + \angle P + \angle C = 180°}$$

And remember the hint recommended that we call $\angle P$ & $\angle C$ each $n°$ each –which we can do because we know they are equal to each other (that's why we can use the same variable for both of them). Sticking "$n°$" in for $\angle P$ & $\angle C$ and solving, we get:

$$138° + n° + n° = 180°$$

$$\rightarrow 138° + 2n° = 180°$$

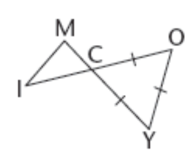
$$\rightarrow 2n° = 42°$$

$$\rightarrow n° = \mathbf{21°}$$

Great - we've learned that the other two angles in the triangle, $\angle P$ & $\angle C$, each equal 21°. And now we have all three!

Answer: 21°, 21° & 138°

4. In the IMCOY diagram, $\triangle COY$ is an equilateral triangle, and $\angle I = 25°$. What is the measure of $\angle M$? *(Hint: vertical angles!)*



If $\triangle COY$ is an equilateral triangle, that means all its angles equal $60°$. For example, $\angle OCY = 60°$. Because vertical angles are always congruent, that means $\angle MCI$ must also equal **$60°$**, right? Let's look at the triangle $\triangle MIC$.
Since we've been told that $\angle I = \mathbf{25°}$, we now have two of the angles in the triangle $\triangle MIC$, and we can solve for the third angle, which is $\angle M$, the angle we're being asked to find!

$\angle MCI + \angle I + \angle M = 180°$

$\rightarrow$ $\mathbf{60°} + \mathbf{25°} + \angle M = 180°$

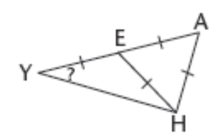
$\rightarrow$ $85° + \angle M = 180°$

$\rightarrow$ $\angle M = 95°$

nice!

Answer: $\angle M = 95°$

5. In the diagram YEAH, $\triangle AEH$ is equilateral, and $\triangle YEH$ is isosceles. What is $m\angle Y$? *(Hint: Fill in all the angles we know, and use supplementary angles!)*



Let's start figuring out what we know… since $\triangle AEH$ is equilateral, we know all its angles are $60°$. That's a lot of information already! Notice that $\angle YEH$ and $\angle HEA$ are supplementary. And since $\angle HEA$ is one of the angles in the equilateral triangle, it equals $60°$. Since they are supplementary, we can write:

$$\angle YEH + \angle HEA = 180°$$

$$\rightarrow \angle YEH + 60° = 180°$$

$$\rightarrow \angle YEH = \mathbf{120°}$$

Great! Now, we've been told that $\triangle YEH$ is isosceles, and judging from the markings on the diagram, its congruent legs are $\overline{YE}$ and $\overline{EH}$, which means that $\angle Y \cong \angle EHY$ (those are the base angles).
And for this little isosceles triangle, we know the sum of its angles equals $180°$, so we can write this equation:

$$\angle Y + \angle EHY + \angle YEH = 180°$$

$$\rightarrow \angle Y + \angle EHY + 120° = 180°$$

$$\rightarrow \angle Y + \angle EHY = 60°$$

But we know that $\angle Y \cong \angle EHY$, so that means they both must equal 30°, right? We were asked to find $m\angle Y$, and that's what we've done! (We also could have labeled $\angle Y$ & $\angle EHY$ with $n°$, like we did in #3, and solved for $n°$.)

Answer: $\boldsymbol{m\angle Y = 30°}$