Girls Get Curves

Understanding Why AAS Works

(As promised in the footnote of p. 122 in *Girls Get Curves*)

In chapter 8 of Girls Get Curves, we saw some great shortcuts for proving that two triangles are congruent: Even if all we know about two triangles is that a *few* corresponding angles and/or sides are congruent, we can often prove that the entire two triangles are congruent! And we saw all sorts of neat ways to visualize why different combinations work (like the sunglasses thing for ASA).

But the AAS shortcut isn't as easy to visualize until we have learned something about similar triangles, like we did in chapter 17. So make sure you're familiar with that stuff before continuing on…

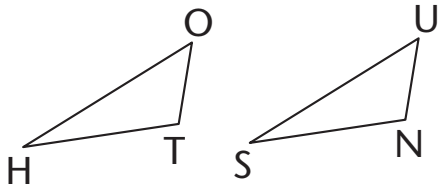
Let's do it! First let's review the shortcut we're talking about…

Angle-Angle-Side Rule (AAS or SAA): For any two triangles, if two pairs of corresponding angles and a side *not* in between are congruent, then the two (entire) triangles are congruent.

Here's why this shortcut works: If we know that two sets of angles are congruent, then we automatically know that the two triangles are similar, because of AA, right? And if two triangles are similar, that means the ratio of their corresponding sides are equal. (See p. 286 to review all of this.)

Then we could ask: Well, when what *is* the ratio of the corresponding sides? Since the "heart" sides above are congruent, that means the ratio of corresponding sides is 1: 1. And since in similar triangles, the ratio of all corresponding sides are equal, that means all corresponding sides' ratios must be 1: 1. In other words, all the corresponding sides are congruent to each other! And now we have SSS between our two triangles, which means the entire two triangles must be congruent. Nice.

For example, let's say we want to prove $\triangle HOT \cong \triangle SUN$, and that we're given $\angle H \cong \angle S$, $\angle T \cong \angle N$, and $\overline{HO} \cong \overline{SU}$. Since we have two congruent sets of angles, we have AA, and that means the two triangles are similar, right? And if two triangles are similar, then by CSSTP, we know the *ratios* between corresponding sides are equal. This means that, for example, $\frac{HO}{SU} = \frac{HT}{SN}$, right? (Find those sides on the triangles)



But since we were given $\overline{HO} \cong \overline{SU}$, that means *HO* = *SU*; in other words: $\frac{HO}{SU} = 1$! And that means that $\frac{HT}{SN} = 1$, too, which means that *HT* = *SN*. Same goes for the third sides, which gives us *OT* = *UN*. So, just one set of congruent corresponding sides on our two similar triangles *forced* our ratio to be 1:1, and then the two entire triangles *had* to be congruent. Now you know why SAA works, and your brain just got a little bigger.

Read this again until you could explain it to someone else… and then *do* explain it to someone else! (Like your teacher – she'll be so impressed..!)

Did I mention how impressed I am, too? ☺