"Hot X: Algebra Exposed"

Supplemental PDF: Proof that $0.\overline{9}=1$

...as promised on p.399

Yep, we're going to prove that $0.\overline{9} = 1$.

(Remember: $0.\overline{9}$ = 0.999999999…. where the 9's go on forever. Say the title of this proof out loud like this: "Point nine repeating equals one.")

Seems a bit strange, right? Like, maybe $0.\overline{9}$ is really close to 1, but never gets there? The key to this is that the 9's go on *forever* – there are an *infinite* number of them. If there were only, say, a billion 9's after the decimal point, then you'd be right; it would be very close to 1 but not equal to it. However, with a little algebra, we can actually prove that an infinite number of 9's means we attain the value of 1. Wacky, but true!

Most teachers don't show you this proof until Algebra II (if ever), but I know you can handle it. Just follow along, and keep an open mind!

Let's start off by welcoming our friend, *x*. You know, that little placeholder whose value we don't happen to know yet? Except let's give her a value; we'll say that she equals this weird repeating thing we're dealing with, so we'll say:

$$x = 0.\overline{9}$$

With me so far? It's a free country, so why not label *x* like this? Next we're going to use a little algebra on the above equation to prove that *x* = 1, and if we do this, then we will have proven that $0.\overline{9}$ = 1, since they both equal little miss "*x*."

I know what you're thinking: "Yeah sure, but how do we use the above equation to prove that *x* = 1?" Have a little faith.

Now just follow along: Ok, so if $x = 0.\overline{9}$, then what would 10*x* be equal to? Well, that's just 10 times *x*. So, that means we'd have to multiply 10 times $0.\overline{9}$, right?

So: $10x = \mathbf{10(0.\overline{9})}$.

And in order to multiply any decimal number by 10, we just move the decimal place to the right one spot (see p.122 in Math Doesn't Suck to review this idea), so:

$$10(0.\overline{9}) = 9.\overline{9}$$

Since there are an infinite number of 9's after the decimal point, when we move the decimal point to the right one place, there are still an infinite number of 9's after the decimal place. Infinity's just like that. Y'know, *infinite*.

So now we know that $\mathbf{10}\boldsymbol{x} = \mathbf{9.\overline{9}}$

So far, so good?

You're probably wondering where I'm going with this… Do you remember on p.169 of *Hot X: Algebra Exposed*, where I showed you that we can subtract equations from each other and get a true statement? If this doesn't sound familiar, go read p.169 right now and then come back. Otherwise, what you're about to see won't convince you of anything!

In this case, the two equations we'll use in the subtraction are the first one we started with: $x = 0.\overline{9}$, and the one we just figured out: $10x = 9.9$. Let's subtract these from each other and see what happens! Whatever we get, we'll know it's another true equation.

Notice below that on the left side, $10x - x = 9x$. And on the right side, $9.\overline{9} - 0.\overline{9}$, the infinite strings of 9's after the decimal points get subtracted away, leaving us with 9:

$$10x = 9.\overline{9}$$

$$x = 0.\overline{9}$$

$$-\ \_\_\_\_\_$$

$$9x = 9$$

So we've discovered a new true statement: $\mathbf{9}\boldsymbol{x} = \mathbf{9}$. Do you see what's about to happen, once we simply this?

Well, just divide both sides by 9 and you'll get $x = 1$! Yep, $x = 0.\overline{9}$ and $x = 1$, which means that $\mathbf{0.\overline{9} = 1}$.

Let me emphasize again; most teachers don't show you this until Algebra II, because the more comfortable you are with all of the algebra concepts we used, the easier this proof is to "believe." But think about it; all we did was label x, then did some multiplication, and then subtracted a system of equations, just like we did throughout Chapter 12 of *Hot X: Algebra Exposed*! So you, missy, have all the tools you need to amaze your friends, parents, and parents' friends with the proof that $0.\overline{9} = 1$.

Here's a presentation tip: Showing this proof to people is kind of like a magic trick, and if you want it to go well, script it out ahead of time and practice it out loud,

making it satisfying and dramatic with well-placed phrases like, “Everything you’ve ever understood about numbers is about to be shattered” and “Are you with me so far?” etc. (You want them to be “with you” for as long as possible, so that they have less to object to when you show them the end of the proof.) Warning: You may make some adults very agitated by this proof because the results go against their intuition, which has been with them for many years longer than you’ve been alive.

Oh, and if you really want to rile up a protesting adult, after you’re finished with the proof, if they are still objecting, then ask them if they “believe” that $0.\overline{3} = \frac{1}{3}$. Usually they will say, “Well, sure, but that’s different.” Then tell them to multiply both sides of that equation by 3, and ask them what they get. Just dare them to tell you they don’t get $0.\overline{9} = 1$. ☺ Have fun!